THE STRENGTH OF THE EARTH'S CRUST

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PART III. INFLUENCE OF VARIABLE RATE OF ISOSTATIC COMPENSATION

INTRODUCTION AND SUMMARY

The work of Hayford on the deflections of the vertical, and of Hayford and Bowie on the anomalies of gravity, has supplied the geodetic data from which future work must start. As an initial basis to guide their work, it was desirable to assume the hypothesis that isostatic compensation was complete for each topographic irregularity, giving local compensation, and that it was uniformly distributed to a constant depth. The actual results may then be compared to this ideal of local, uniform, and complete isostasy and the degree of departures noted, as given by residuals and anomalies.

In Part II the subject of the regional distribution of compensation was examined and the conclusion was reached that the crust was sufficiently rigid to bear such mountains as Pikes Peak without requiring special compensation below. In general it is thought compensation in mountain regions extends to more than 200 km. and in some regions to more than 400 km. In this part are considered the effects of variations in the vertical distribution of

compensation and the degree to which such variability may give rise to anomalies and residuals without signifying incompleteness of compensation in the column as a whole or regional departures from isostasy.

In order to show the limits of variation in density which are to be expected, the specific gravity of rocks is first considered. Figures are computed for the mean specific gravity of igneous rocks and the three types of sediments. It is shown that the range of variation is an important factor. Under the subject of the relations between mass and the distance of mass upon anomalies, the effects are computed of unit masses at various depths and extending various distances.[1] This lays the basis for considering the influence of the specific gravity of the surface geologic formations upon the difference between the mean anomalies for stations on pre-Cambrian and those on Cenozoic areas. It is found that the greater density of the older rocks accounts for a part and another part is accounted for by their resistance to erosion. This still leaves, however, large outstanding regional variations not related to surface geology or topography and requiring some other explanation. To that end criteria are discussed for the recognition and separation of the effects of mere variable vertical distribution of compensation on the one hand, from partial regional absence of isostasy on the other. It is concluded from the application of these criteria that the anomalies are in large part caused by real regional departures from isostasy extending over broad areas. The results are thus

[1] A paper by Gilbert has recently appeared entitled "Interpretation of Anomalies of Gravity" (*Part C, Professional Paper 85, U.S. Geological Survey*, 1913). This did not reach the present writer until after Parts III and IV of this article were in galley proof, so that his results cannot be as fully interwoven into the discussion as would otherwise have been the case. On pp. 30, 31, Gilbert considers the interpretation of anomalies on the assumption of vertical heterogeneity of the crust and shows clearly that moderate variations of density in a vertical direction could explain them. From this he infers that the anomalies may be due in part to such irregularities. This is the topic which is treated in Part III of the present article under the title "Interpretation of Anomalies in Terms of Mass and Depth." The method of reasoning is somewhat different, but although the conclusion reached is the same, the calculations given here are intended to bring out in addition the limitations of area and mass within which that principle applies. It is concluded as a result of the following examination of the evidence that although vertical variations of density are a real cause they are not the major cause of anomalies.

confirmatory of those reached in Parts I and II. In addition, however, it appears that there is a regional departure from isostasy of two orders of magnitude. Loads under the mean value, giving anomalies below 0.018 to 0.020 dyne and estimated to be equivalent to about 750 feet of rock, can be carried over regions of irregular boundaries ranging up to from 1,000 to 2,000 km. across. Over such a broad region the anomalies are of one sign except for some smaller well-defined sub-areas of high anomaly within them which may or may not have the same sign. These smaller areas give a higher order of stress magnitude and are of more restricted dimensions, being measured in hundreds of kilometers. They range in magnitude of anomaly to several times the value of the mean and the equivalent radii of their areas probably average 100 to 200 km. The deflection residuals show by the limits of the areas of like sign that the regional variations of gravity anomalies of this areal magnitude extend over the whole country, but where the amounts of the local anomalies are less in value than the mean they are largely masked on the contour map of gravity anomalies (Fig. 5), because of their superposition upon the broader areas. Presumably a multiplication of the gravity stations would bring them to light as undulations in the contours which show the regional departures.

A final conclusion on the subject of the variable vertical distribution of mass must, however, be deferred until consideration has been given to a hypothesis advanced by Gilbert in his recent paper, that heterogeneities of mass below the zone of compensation may be the cause in major or minor part of the apparent departures from isostasy. This is a subject too large to be considered in this third part of the present article, but it is planned to investigate it in Part V by a method of graphic analysis devised for determining the depth of excesses or deficiencies of mass.

THE SPECIFIC GRAVITY OF ROCKS

For a knowledge of the variations of density likely to occur in rocks it is important to know the range in specific gravities shown by the common rock types. The following figures, except those for shale, are taken from Pirsson's *Rocks and Rock Minerals:*

TABLE V

Rock	Specific Gravity
Granite	2.63–2.75
Syenite	2.6 –2.8
Diorite	2.8 –3.1
Dolerite	3.0 –3.3
Limestone	2.6 –2.8
Sandstone	2.5 –2.7
Shale	2.4 –2.8
Slate	About 2.8

[The specific gravity of shale, although the most abundant of sedimentary rocks, is not given in any of the manuals of geology, but Professor Hobbs, who has read much of this manuscript and to whom the writer is indebted for a number of suggestions, has called attention to the above figure as given by Trautwine. In general, Trautwine and Kent give a somewhat greater range in specific gravities and they average a little lower than those here given. The figures from Pirsson, however, probably express more closely the relation of the petrologic type and the more compact states of rocks to their density. They are, therefore, thought to be better representative of the lithosphere.]

These figures show that notable departures may occur from the mean density of the outer crust and suggest furthermore that 2.67, the mean density used by Hayford, is lower than the actual mean. A more thorough analysis of the subject is therefore needed.

The abyssal igneous rocks and metamorphic rocks are almost without pore space. The sedimentary rocks, on the other hand, possess abundant pore space in their unconsolidated states, very little in their compact states. The latter is the usual mode of occurrence in the older geological formations. The density is therefore a function of both mineral composition and porosity. The chemical compositions of the several rock types and also of the average sediment and the average igneous rock are well known. The mineral compositions are less well known but may be computed with a fair degree of accuracy; the densities, on the contrary, are least commonly reported and the mean densities of the rock types cannot in consequence be closely determined by averaging numerous determinations, as is done for the chemical compositions. It seems desirable, therefore, to compute the densities of the rock types from the chemical and mineral compositions, combining this with the densities of the individual minerals, making a separate correc-

tion for the porosity factor. The data, assembled from various sources[1] and subjected to computation, give the following results:

TABLE VI

COMPOSITION OF AVERAGE IGNEOUS ROCK

Mineral	Percentage
Quartz	12.0
Feldspars	
Orthoclase molecule	22.0
Albite molecule	29.5
Anorthite molecule	8.0
Hornblende and pyroxene	16.8
Mica	3.8
Accessory minerals	7.9
	100.0

TABLE VII

COMPOSITION OF AVERAGE SEDIMENTS

Mineral	Shale	Sandstone	Limestone
Quartz	22.3*	66.8*	2.0
Feldspars			
Orthoclase	18.0	7.0	0.3
Labradorite	12.0	4.5	0.1
Clay	25.0†	6.6†	2.0‡
Limonite	5.6	1.8	0.6
Calcite	5.7	11.1	55.0
Dolomite			35.0
Other minerals	11.4	2.2	5.0
	100.0	100.0	100.0

* The total percentage of free silica.

† Probably sericite in part; in that case the feldspar figure becomes lower.

‡ Two per cent clay takes 0.79 of Al_2O_3. This requires that most of alkalies form non-aluminous hydrous silicates or that 0.81 Al_2O_3 as given by Clarke is too low.

It is thought that the densities without porosity are figures of some value for geodetic computations. The chief error in making the final estimates is in connection with the lack of accurate knowledge regarding the pore space of those sedimentary rocks not used

[1] For data on the mean chemical and mineral composition of rocks see F. W. Clarke, "Data of Geochemistry," *Bull. 491, U.S. Geol. Surv.*, 1911, pp. 30, 31. For specific gravities of minerals see Pirsson, *Rocks and Rock Minerals*, 1908, p. 31; also Dana, *Mineralogy*. For a discussion of pore space see Fuller, "Total Amount of Free Water in the Earth's Crust," *Water Supply Paper No. 160, U.S. Geol. Surv.*, 1906, pp. 59–72.

as building stones, but this affects appreciably the density of only a superficial layer and chiefly of the youngest deposits.

The ratio of shale, sandstone, and limestone in the average sediment in percentage is, according to Mead,[1] shale 80, sandstone 11, limestone 9. The ratio of average porosities in percentage is, according to Fuller,[2] crystalline rocks 0.2, shales 4, sandstones 15, limestones 5. The figure given by Fuller for shale rests upon a single determination of 7.8 per cent by Delesse, and is averaged in by Fuller with slate. Eight per cent porosity will here be assumed as probably a better estimate. This gives the porosity of the average sedimentary rock as 8.5 per cent. The pore space may be taken, following Fuller's estimate, as half filled with water.

From these data the specific gravities are computed to be as follows:

TABLE VIII

SPECIFIC GRAVITIES COMPUTED FROM MINERAL COMPOSITIONS

Rock	No Pore Space Allowed	Pore Space Half Filled with Water
Average igneous rock..	2.80	2.80*
Shale...............	2.69	2.51
Sandstone...........	2.67	2.35
Limestone...........	2.76	2.64
Average sedimentary rock..............	2.70	2.50

*The same figure as used by Chamberlin and Salisbury, *Geology*, I (1904), 538; also by Pirsson, *Rocks and Rock Minerals*; also by G. H. Darwin as the density of the outer crust.

Where Cenozoic deposits occur in thickness, they are considerably compacted except at the surface, but still the mean specific gravity, owing to the abnormal pore space and deficiency in limestones, is doubtless less than 2.50; 2.45 may be taken. It is probable, on the other hand, that the Paleozoic rocks on the whole have somewhat less pore space than this average, especially as the porosity figure for sandstone rests mainly upon determinations for brownstone, a rather porous type; 2.55 may then be taken as the average for Mesozoic and Paleozoic formations. The pre-Cambrian

[1] "Redistribution of the Elements in the Formation of Sedimentary Rocks," *Jour. Geol.*, XV (1907), 238–56.

[2] *Loc. cit.*

rocks contain both igneous and sedimentary formations, but the considerable iron ore and metamorphic nature would bring the specific gravity of the sediments somewhat above the average of 2.70 for non-porous sediments. Broad areas of pre-Cambrian probably range therefore between 2.75 and 3.00 in specific gravity. More limited areas, because of a predominance of granite and quartzite, may range as low as 2.70. About 2.67, however, would be a minimum.

As these are merely averages it is better in basing calculations upon them to assume a certain range in density for each figure and to obtain thus a knowledge of the influence of reasonable variations upon the results. The data may then be tabulated as follows:

TABLE IX

ESTIMATED MEAN SPECIFIC GRAVITIES OF GEOLOGIC FORMATIONS

Pre-Cambrian	2.75–2.80
Paleozoic and Mesozoic	2.50–2.60
Cenozoic	2.40–2.50

The range in these specific gravities shows the necessity of considering them in all refined calculations on the anomalies of gravity. In place, however, of using a mean density figure for all stations on formations of a certain geologic age, it would be of much more value to have measurements of the actual surface densities occurring in each area; also estimates by geologists, based on geologic structure and these surface measurements, of the densities extending to the base of the sedimentary rocks of each locality.

It seems probable from the mean density of 2.80 obtained for igneous rocks that the density of 2.67 used by geodesists for the mean density of the zone of compensation is too low. If any variation from the average composition takes place with depth within the limits of 76 miles, it is likely to be a variation toward more basic and heavier rocks. Assuming, however, an average uniformity of chemical composition, the opposing effects of temperature and pressure remain to be considered. Using the coefficient of expansion of the average igneous rock computed by W. H. Emmons,[1] 0.000,019,9 for 1° C., and a temperature gradient of

[1] Chamberlin and Salisbury, *Geology*, I (1904), 547.

1° F. for 60 ft. in depth, gives an aggregate expansion of 3.6 per cent to the outer 76 miles. Using 6,500,000 as the modulus of cubic compressibility of the average rock in pound-inch units[1] gives a total compression of 3.7 per cent to the outer 76 miles due to pressure; that is, the volume effects of heat and pressure practically offset each other within the zone of isostatic compensation. Therefore 2.80 appears to be the lowest mean figure which should be taken. The use of 2.67 as a mean figure requires for isostatic equilibrium a density of but 2.60 extending to a depth of 76 miles under land 3 km. high, a figure lower than the specific gravity of granite.

INTERPRETATION OF ANOMALIES IN TERMS OF MASS AND DEPTH

Suppose that the zone of isostatic compensation is not ot uniform density under any one station, but contains masses of variable density irregularly distributed. Let these masses be of considerable thickness and area as compared to the depth of the zone of compensation. Suppose that the topography is so adjusted to the aggregate density that the pressures are everywhere equal at the bottom of the zone of compensation. Abnormally light masses would then have to be balanced by abnormally heavy masses in the same column. There would still be deflections of the vertical and anomalies of gravity because gravitation varies inversely with the square of the distance, the upper and adjacent masses of abnormal density affecting the station more than those more distant ones of opposite abnormality lying vertically below the upper. The residuals from deflection and gravity measurements would under such an arrangement measure strains within the outer crust but not upon its bottom. The strains, if produced by abnormalities in the upper parts of the crust, would further be proportionately smaller and yet give rise to residuals of a certain magnitude than if produced by abnormalities in the lower parts of the crust. This aspect of the problem must be investigated before any final significance regarding the strength of the crust can be attached to the grouping of residuals discussed under the

[1] F. D. Adams and E. G. Coker, *An Investigation into the Elastic Constants of Rocks, More Especially with Reference to Cubic Compressibility*, 1906, p. 67.

last part of Part II. It leads to a consideration of the relations between mass, distance, and anomaly.

Under the title of "Interpretation of Anomalies in Terms of Masses"[1] Hayford and Bowie show that the excesses and deficiencies of mass to a great distance have an effect upon the gravity anomalies and that therefore the guarded expression "net effective excess (or deficiency) of mass" is necessary for correctness. They give the following tabulation to show the influence of uncompensated masses in the crust in giving gravity anomalies when the gravity is computed on the assumption of isostasy:[2]

TABLE X

Each tabular value is the vertical attraction in dynes produced at a station by a mass equivalent to a stratum 100 ft. thick, of density 2.67, and of the horizontal extent indicated in the left-hand argument, if that mass is uniformly distributed from the level of the station down to the depth indicated in the top argument and from the station in all directions horizontally to the distance indicated in the left-hand argument.

Radius of Mass	Depth				
	1,000 Ft.	5,000 Ft.	10,000 Ft.	15,000 Ft.	113.7 Km.
1,280 m. (the outer radius of zone E)	0.0029	0.0018	0.0011	0.0008	0.0000
166.7 km. (the outer radius of zone O)	0.0037	0.0034	0.0034	0.0034	0.0024
1,190 km. (or 10°40′, the outer radius of zone 10)	0.0040	0.0037	0.0037	0.0037	0.0034

On p. 111 it is concluded by these authors that the best working hypothesis is to take

each 0.0030 dyne of anomaly as due to an excess (or deficiency) of mass equivalent to a stratum 100 ft. thick. This working hypothesis is equivalent, as may be seen by inspection of the table just given, either to the assumption that the excess (or deficiency) of mass is uniformly distributed to a depth of 113.7 kilometers and extends to a distance of more than 166.7 kilometers and less than 1,190 kilometers from the station, or that it extends to a distance of 166.7 kilometers from the station and is distributed to an effective mean depth of more than 15,000 feet and less than 113.7 kilometers, or the working hypothesis may be considered to be a combination of these two assumptions.

The mean anomaly of 0.018 dyne, interpreted on this basis of 0.030 dyne being taken as equivalent to 100 ft. of mass, gives a

[1] Hayford and Bowie, p. 108. [2] *Ibid.*, 1912, p. 109.

mean departure from isostatic compensation amounting to 600 ft.; given more exactly by Bowie as 630 ft.

It is seen from the quoted statement that the authors accept, first, as one alternative a very widespread regional net excess (or deficiency) of mass uniformly distributed in depth; or, second, a somewhat broad regional distribution but confined to the outer part of the zone of compensation; or, third, some combination of the two assumptions.

The first assumption would throw a real strain upon the bottom of the zone of compensation and signifies regional compensation to limits very far beyond those stated elsewhere by the authors. It is therefore inconsistent from that standpoint, but gives a smaller vertical load and consequently a smaller vertical departure from the level giving isostatic equilibrium than would a more limited area. If, for example, it be assumed that the radius of the zone limiting regional compensation is 58.8 km., which is about the maximum limit for regional compensation which Hayford allows elsewhere; then it may be computed that for uniform distribution of the excess (or deficiency) of mass to a depth of 114 km., a mass equivalent to 100 ft. of density 2.67 corresponds to an anomaly of but 0.0013 dyne instead of 0.0030. This would, for a mean anomaly of 0.018, signify an average departure over the United States of 1,380 ft. from the level giving isostatic equilibrium, instead of 600 ft.

The second assumption, that the excess (or deficiency) is in the outer part of the crust, gives also a much higher anomaly for a unit mass than would an equally permissible assumption that the excesses or deficiencies occurred at various levels and on the average were at a depth of one-third or one-half of the zone of compensation. The relationship of anomalies to geologic formations, to be discussed later, shows certain variations in density in the outer crust, but the greater parts of the anomalies are not due to this cause. From the previous discussion on the limits of regional compensation it would seem that, on the assumption that the excesses or deficiencies of mass are on the whole uniformly distributed, 0.0024 would be an appropriate figure to use as the mean anomaly for unit thickness of mass. The highest anomalies, however, are

probably better interpreted by 0.0030 as a divisor, since as a class they must be assumed as due to excesses or deficiencies of mass which are both near and large. This does not mean, however, that the larger masses are not assumed as scattered uniformly, according to the laws of chance, through the crust. It is seen, then, that Hayford and Bowie have favored those interpretations which gave a large anomaly per unit mass and have ascribed the total anomaly as on the average to be interpreted on this basis, obtaining thereby a smaller figure as the mean departure in feet from the level for perfect compensation. They have not discussed, furthermore, in the text the influence of deeper-seated variations of density, which might give considerable residuals, nor the possibility that departures from the mean density in opposite directions might balance each other so as to give equal pressures at the bottom of the zone of compensation. The latter case will not seem improbable to the geologist. The great batholiths of the Archean appear to make a universal floor in the crust. They range in composition from granites to gabbros and have come to rest at various levels. Light and heavy masses may well be irregularly distributed in the same vertical cylinder. If at the time of origin the whole were too heavy, a tendency would have arisen for the column to sink until equilibrium was attained. If the whole, on the contrary, were too light, the column would have tended to rise until a heavier base balanced the lighter mass above. Thus, if irregular distribution of density arose as the result of vertical igneous intrusion, the whole region would tend to seek that level where the irregularities would balance.

In order to gain quantitative ideas as to this possibility of partly explaining the anomalies, the writer has made calculations on the following assumptions. A station is situated upon the axis of a vertical cylinder extending from the station to a depth of 114 km. The radius is taken successively at 58.8, 166.7, and 1,190 km. Let such a cylinder be divided into five equal cylinders by horizontal planes. Let each of the five be equivalent in mass to a cylinder of the same radius but only 100 ft. in depth and of density 2.67; in other words, the unit mass as used by Hayford and Bowie. What will be the attraction in dynes per gram pro-

duced at the station by each cylinder respectively?[1] The results are as follows:

TABLE XI

VERTICAL ATTRACTION IN DYNES ON ONE GRAM AT STATION BY CYLINDER 22.8 KM. THICK, DENSITY 0.00357, EQUIVALENT IN MASS TO THICKNESS OF 100 FT. AT DENSITY 2.67

No. of Cylinder	Depth in Km. from Station to Top of Cylinder	Attraction for Radius of 58.8 Km.	Attraction for Radius of 166.7 Km.	Attraction for Radius of 1190 Km.
I	0.0	0.0031	0.0032	0.0036
II	22.8	0.0017	0.0028	0.0035
III	45.6	0.0010	0.0024	0.0035
IV	68.4	0.0007	0.0020	0.0035
V	91.2	0.0005	0.0017	0.0034

The results for radius 58.8 km. show that masses of this size situated near the bottom of the zone of compensation exert but a fraction of the influence given by equivalent masses near the surface. A balancing of light and heavy masses in a column of this radius would give isostasy at the base and yet produce notable anomalies. For radius 166.7 km. the importance of depth is much diminished. For radius 1,190 km. it practically disappears. This means that a wide regional variation in depth with plus and minus departures from the uniform density, the light and heavy layers balancing, would not produce anomalies provided, as stated, there was isostatic equilibrium at the base.

To give a somewhat extreme illustration; suppose that the upper cylinder, I, is 2 per cent lighter than the mean density of

[1] The formula for making these computations was kindly worked out for me by Professor H. S. Uhler, checking it as given by B. O. Pierce, *Newtonian Potential Function*, p. 8. It is as follows:

$$F=2\pi\rho\gamma\,[\sqrt{a^2+c^2}-\sqrt{a^2+(c+h)^2}+h].$$

in which

F = force in dynes per gram.
ρ = density, in this case = 0.003,57.
γ = constant of gravitation = 0.000,000,066,58.
a = radius of cylinder.
c = distance on axis from station to top of cylinder.
h = depth of cylinder; in this case 22.8 km.

For radii of 58.8 and 166.7 km. no correction need be made for curvature of the earth's surface. For a = 1190 km. an empirical correction was obtained by comparing the results with Hayford's computations.

The writer overlooked until later the fact that Hayford and Bowie also give this formula with a different notation on p. 17 of their work.

2.67 and the lower cylinder, V, is 2 per cent heavier. Let these abnormalities be limited areally to the cylinder. This is a departure in density of 0.054, 15.1 times the density 0.00357. The anomalies will be as follows:

TABLE XII

ANOMALIES DUE TO IRREGULAR VERTICAL DISTRIBUTION OF DENSITY

No. of Cylinder from Table	Density 2 Per Cent from Mean	Anomalies		
		Radius 58.8 Km.	Radius 166.7 Km.	Radius 1190 Km.
I.	2.616	−0.047	−0.048	−0.054
V.	2.724	+0.008	+0.026	+0.051
Resultant anomaly.		−0.039	−0.022	−0.003

It is seen from this tabulation that, first, irregular superposed but balanced positive and negative distributions of density up to distances as large as the radii of the areas of grouped residuals could produce at least a considerable part of the anomalies; or, second, actual departures from isostatic equilibrium with the resultant strain on the crust could produce them; or, third, a combination of the two. In the second case, as Hayford and Bowie show,[1] the anomalies could result from a layer a few miles thick adjacent to the station and of very abnormal density; or from deep and regional masses of great volume, but departing only slightly from the mean density. The choice between these several alternatives, or the degree to which they co-operate, must be investigated under the following topics.

RELATIONS OF ANOMALIES TO EXPOSED GEOLOGIC FORMATIONS

The latest data given by Bowie on this subject are shown in Table XIII (p. 222):[2]

These figures of course are not to be regarded as of high precision, as may be seen by comparing the earlier and later results.

[1] *Op. cit.*, Pp. 108–11.

[2] "Some Relations between Gravity Anomalies and the Geologic Formations in the United States," *Am. Jour. Sci.* (4), XXXIII (1912), 237–40.

Hayford and Bowie in their successive publications give the following for the pre-Cambrian and Cenozoic stations, the two groups

TABLE XIII

Geologic Formation	Number of Stations	Mean with Regard to Sign	Mean without Regard to Sign
Pre-Cambrian	10	+0.016	0.026
Paleozoic	31	−0.003	0.019
Mesozoic	20	+0.002	0.015
Cenozoic	29	−0.008	0.021
Intrusive and Effusive	11	−0.007	0.015
Unclassified	22	+0.011	0.020
All stations	123	0.000	0.019

to which the attention will be confined. A few stations of high anomaly must have considerable influence on the result, as most of the stations are used in common in all of the estimates.

TABLE XIV

	Geologic Formation	Number of Stations	Mean with Regard to Sign	Mean without Regard to Sign
Hayford and Bowie, U.S.C. and G.S.	Pre-Cambrian	7	+0.019	0.026
	Cenozoic	20	−0.011	0.021
Bowie, U.S.C. and G.S.	Pre-Cambrian	9	+0.024	0.024
	Cenozoic	33*	−0.007	0.021
Bowie, *Am. Jour. Sci.*	Pre-Cambrian	10	+0.016	0.026
	Cenozoic	29	−0.008	0.021

* Fifteen stations have plus anomalies, 17 have minus anomalies.

Bowie's figures in the *American Journal of Science* will be used in the following discussion.

Bowie favors the explanation that these relations of anomalies to geologic formations are due to slight changes of density extending more or less through the zone of compensation and leading to departures from perfect isostasy. The writer, however, is led to favor the view that about one-half of the contrasted anomaly for these two groups is due to a lesser density within the outer mile of crust beneath the Cenozoic stations, as contrasted to the outer mile of crust beneath the pre-Cambrian stations. The remainder of the anomaly it is thought is explained by the ease of erosion of Cenozoic formations, the resistance to erosion of the pre-Cambrian

rocks. The latter consequently tend to stand above the regional levels. They therefore possess surficial excess both of density and volume.

The average thickness of sedimentary rocks if spread uniformly over the globe is thought to be between 2,000 and 2,500 ft.[1] Over the pre-Cambrian areas it must average much less; over the areas of later formations much more. Under the Cenozoic stations assume:

1,000 ft. of sediments at density...........	2.40 to 2.50
4,000 ft. of sediments at density...........	2.50 to 2.60
Giving a total of 5,000 ft. at density........	2.48 to 2.58
With a deficiency of density of............	0.19 to 0.09

Under the pre-Cambrian stations assume:

5,000 ft. of crystalline rock at density......	2.75 to 2.80
An excess of density of..................	0.08 to 0.13

This does not involve the improbable assumption that below the outer 5,000 feet of crystalline rock of density 2.75 to 2.80 the density suddenly decreases to 2.67 and then remains constant throughout the zone of compensation. The vertical density gradient, *if uniform for all points*, has but little effect, it being the *horizontal* variations of density which enter into the problem of isostasy. To maintain conformity with Hayford's figures, therefore, the density 2.67 will be frequently assumed as the mean density of the lithosphere, although the previous discussion shows that it cannot be assumed as the density of the outer mile of crystalline rocks when comparing these to the mile of sedimentary rocks taken as the mean depth underlying the Cenozoic stations.

In comparison with this thickness of 5,000 ft. the average area of formations is very great. A plane sheet of rock 100 ft. thick and of density 2.67, if of indefinite extent, will produce an anomaly of 0.0034 dyne upon a point outside of it, irrespective of the distance to that point. This theory may be applied without gross error to the relation of surface geologic formations to anomalies. If this unit mass be expanded from 100 to 5,000 ft. thickness, the

[1] F. W. Clark, "Data of Geochemistry," *Bull. 491, U.S. Geol. Surv.*, 1911, p. 30.

density will be decreased to 0.053 that of water. The data may then be tabulated as follows:

TABLE XV

COMPUTED ANOMALIES DUE TO DENSITIES OF SURFACE FORMATIONS

	Deficiencies or Excesses of Density	Anomalies in Dynes per Gram Due to Thickness of 5,000 Ft.
Unit mass...........	0.053	0.0034
Cenozoic............	−0.19	−0.012
	−0.09	−0.006
Pre-Cambrian.......	+0.08	+0.005
	+0.13	+0.008

These mean anomalies of the pre-Cambrian due to the greater density of the outer 5,000 ft. of rock, when compared to the Cenozoic anomalies, are, as shown by this tabulation, at a minimum 0.011 greater, at a maximum 0.020 greater, at a mean 0.0155 greater. The difference of the means shown by geodetic measurement was 0.024. The specific gravities seem to have been taken as far apart in limits as is allowable and the assumed mean thickness of sediments as 5,000 ft. beneath the Cenozoic stations is a generous figure; the mean thickness is more likely to be less, rather than greater. The means for the geodetic anomalies as related to geologic formations are perhaps subject to about the same degree of error as the determinations of the anomalies from the specific gravities and thickness. The result, although not of a high order of accuracy, shows that although the range in specific gravities accounts for a considerable part, perhaps one-half or two-thirds, of the relation of anomalies to geologic formations, it can hardly account for the whole.

To find the cause for the remaining portion of the anomaly, two hypotheses may be considered: first, that it is due to a slight regional excess of density extending to a depth of 114 km., the hypothesis favored by Bowie; or, second, that the Archean areas on the average stand higher than the Cenozoic by virtue of resistance to erosion.

The geologic evidence as it is at present understood is against the first hypothesis and in favor of the second. This statement

is based on the view that Archean and Proterozoic areas have tended to be rising elements of the continent. Erosion instead of sedimentation has been dominant in later geologic time, which is the reason why these rocks are now exposed as surface formations. If there is any deep-seated departure of density from the mean this tendency to rise should correspond, however, to a deficiency of density persisting through the geologic ages, extending through much of the zone of compensation and offsetting the more than average surface density. Such a regional deficiency is opposite in character to the excess which is postulated by Bowie as an explanation of the positive anomalies.

Assume then as the next step in the argument that the density of the zone of compensation beneath the pre-Cambrian areas to a depth of 114 km. is the same as under Cenozoic areas except for the outer 5,000 ft., both having a mean density of 2.75 to 2.80, but taken here as 2.67. The outstanding anomaly in that case is due to a longer mean column for the pre-Cambrian areas and consequently greater mass above the level of complete compensation. If the mean radius of these longer pre-Cambrian and shorter Cenozoic columns is as great as 166.7 km., then the unit excess or deficiency of mass of 100 ft. at density 2.67 when spread over these columns will correspond to an anomaly of 0.0024. If the mean effective areas of the pre-Cambrian and Cenozoic formations affecting individual stations are less, the unit mass will give a smaller unit anomaly. If the mean effective areas are greater, the unit anomaly will not, however, rise above 0.0035. Assume then in conclusion a mean radius of 166.7 km., an anomaly of 0.0024 dyne as resulting from 100 ft. of added mass of mean density, and the outstanding anomaly not accounted for by the surficial densities but due to an outstanding difference in volume as between 0.008 and 0.012. These figures correspond to a differential mean elevation of 330 to 500 feet of the pre-Cambrian above the Cenozoic, due to erosion. To physiographers such a conclusion will seem quite in accord with the geologic evidence testifying to the resistance of pre-Cambrian formations.

The character of the Archean and Proterozoic anomalies enters into the problem of crustal rigidity in the following way. If there

were local and close compensation, then as erosion removed the softer surrounding rocks there should be isostatic upwarping of such areas of denudation and relative downwarping of the uneroded crystalline areas. Such warping of the Mohawk, St. Lawrence, and Champlain valleys with respect to the Adirondacks has not been noted, though the problem from the standpoint of field evidence has not been fully studied. The physiographic evidence that residual mountain masses known as monadnocks or unakas have not been shown, however; to be marked by local downwarping and, on the contrary, certainly stand in relief due to circumdenudation, combines with the geodetic evidence of the average excess of gravity for the resistant areas of pre-Cambrian formations, to suggest effective rigidity against the stresses produced by erosion. The evidence, however, as developed thus far from the geodetic standpoint shows that there are more important factors than that of the surface geologic formation, since the larger anomalies are much greater than these figures which have been discussed and hold but little relation to either relief or surface geology. In fact Hayford and Bowie do not find any discoverable relation between the anomalies in general and the topography.

It is thought by the writer, however, that if stations were located especially to test the intensity of gravity over various broad plateaus remaining by circumdenudation and the intensity compared with that over adjacent broad areas of lower level, the mean differential anomalies due to the surface excess of mass in the plateau over the lowlands would rise to a larger figure than the 0.008 to 0.012 dyne which has remained to be explained in the present discussion. These figures are low because certain pre-Cambrian areas, like those in the vicinity of Baltimore and Washington, have been lowered by prolonged denudation and do not stand markedly above the level of younger formations. Furthermore, the tendency of broad pre-Cambrian areas to stand above sea-level is very probably of an isostatic nature. This implies under such areas a slightly lower mean density to the whole zone of compensation which would diminish the anomaly due to the surface elevation. In individual areas of 100 to 200 km. radius, however, such a relation of positive anomaly to pre-Cambrian

formations and plateaus of circumdenudation may not be found, since it is clear that the anomaly from this cause may be much more than neutralized by other causes. A large number of stations covering broad areas would therefore be required adequately to eliminate these other influences from the means.

LARGE OUTSTANDING ANOMALIES NOT RELATED TO GEOLOGY OR TOPOGRAPHY

In Fig. 5, of Part II, the anomalies are shown for all stations in the United States. It is seen that they possess an areal gradation in magnitude which permits the drawing of anomaly contours. The excessive anomalies of both signs cover oval areas in various parts of the country and show a common disregard of physiographic provinces, structural provinces, and geologic formations. Looking at Fig. 5, one cannot see in either the distribution of anomalies or trends of contours a reflection of Atlantic Coastal Plain, or Appalachian Mountains, or Mississippi Valley.

Typical examples of the lack of necessary relation of the large anomalies to geologic formations are seen in the following tabulation:

TABLE XVI

No.	Station	Geologic Formation	Anomaly
123	Albany, N.Y.	Cambro-Ordovician	−0.043
74	St. Paul, Minn.	Cambro-Ordovician	+0.059
96	Mena, Ark.	Pennsylvanian	−0.052
101	Helenwood, Tenn.	Pennsylvanian	+0.040
53, 56	Seattle, Wash.	Quaternary	−0.093
112	Olympia, Wash.	Quaternary	+0.033

The lack of relation of these anomalies to topography is equally striking. It is clear then that internal conditions in the crust, not expressed on its surface, must be the principal cause of these larger departures from isostasy. The large anomalies show their relationship to internal causes most clearly, but the smaller anomalies may also by analogy be ascribed in part to such hidden causes. The results, however, of surface activities—circumdenudation, sedimentation, tangential pressure, or extravasation—must show in large ratio over regions where the internal variations from uniform density are small; but over the greater part of the United States

the distribution of anomalies appears to depend more upon the internal than upon the external departures from regional uniformity and complete isostasy. The internal heterogeneities of mass are therefore presumably greater than the shiftings of mass due to external activities.

CRITERIA FOR SEPARATING VERTICALLY IRREGULAR COMPENSATION FROM REGIONALLY INCOMPLETE COMPENSATION

Suppose the topography smoothed out to a mean level over areas as large as the limits for regional isostasy. The deflection residuals and gravity anomalies would then be due to one or more of three internal causes; first, vertically irregular or laterally displaced compensation; second, regionally incomplete compensation above the bottom of the zone of compensation because of the effective rigidity of the crust above that level; third, regionally incomplete compensation above a certain level because the zone of compensation may be deeper in places, transferring stresses into a deeper rigid earth. The existence of a general approach toward compensation and away from absolute rigidity suggests that the last is not so important as the first two causes. Under this section then will be considered these two causes, their effects upon the deflections of the vertical and the intensity of gravity, with the purpose of drawing criteria by which the action of the two causes may be recognized and separated. To do this it will be necessary to discuss here to some extent the theory of the attraction of underground masses upon stations at the surface of the earth. It has been shown that balanced irregularities in the vertical distribution of densities through the zone of compensation could give pronounced anomalies without disturbing the isostatic equilibrium at the bottom of the zone, since the total weight of the column could still be normal. To show the effect of such balanced irregularities upon a point outside of the column:

Take a vertical line and a horizontal line which intersect. The masses whose effects are to be investigated will be distributed on the vertical line. The effects are to be determined for points on the horizontal line. To express the trigonometric relations between any point on the vertical and any point on the horizontal line, let a point on the vertical line at depth D be defined as at a vertical

angle θ below a point on the horizontal line; the latter to be defined as at distance R from the intersection.

Let the gravitative attraction of unit masses along this vertical line upon any other point either in or outside of this line be represented by F. The horizontal component will be the force producing deflection of the vertical and may be represented by Fh. The vertical component will give the acceleration of gravity due to the unit mass and may be represented by Fv. Taking the unit mass such that the constants will have a value of unity, the following relations are deduced:

Attraction of unit mass at depth D, upon a point at R:

$$Fh=\frac{\cos^3\theta}{R^2}$$

$$Fv=\frac{\tan\theta\cos^3\theta}{R^2}$$

For the intersection point,

$$R \text{ and } \theta=O \text{ and}$$

$$Fh=0$$

$$Fv=\frac{1}{D^2}$$

Let the depth of the zone of compensation, 114 km., be taken as unit distance, 1.00, and for purposes of discussion let points I, II, III, IV be located on a vertical line at depth of 0.25, 0.50, 0.75, and 1.00 as shown on Fig. 6. Solving the equations for these points and for various values of R gives the following tabulation:

TABLE XVII

Table of Relative Attractions

(Not in dynes per gram)

Attraction by Unit Masses at			Attraction at Stations for Various Values of R									
No.	Depth	Angle below $R=1.00$	$R=0$		$R=0.25$		$R=0.50$		$R=1.00$		$R=2.00$	
			Fh	Fv	Fh	Fv	Fh	Fv	Fh	Fv	Fh	Fv
0..........	0	0	0	0	16.00	0	4.00	0	1.00	0	0.25	0
I..........	0.25	14°02′	0	16.00	5.60	5.60	2.88	1.44	0.91	0.23	0.24	0.03
II.........	0.50	26°34′	0	4.00	1.44	2.88	1.40	1.40	0.72	0.36	0.23	0.06
III........	0.75	36°52′	0	1.78	0.51	1.52	0.68	1.04	0.51	0.38	0.21	0.08
IV........	1.00	45°00′	0	1.00	0.21	0.91	0.36	0.72	0.35	0.35	0.18	0.09

Fig. 6 shows the curves for $R=1$. For any other value of R the curves would be the same in form, but the scales of ordinates and abscissas would be changed. These curves may be used therefore in a general way.

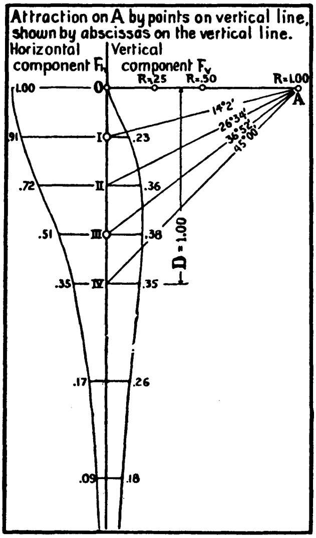

FIG. 6

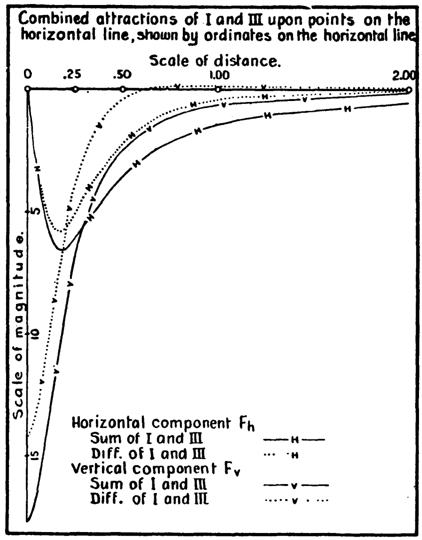

FIG. 7

FIG. 6.—Curves showing relative attraction of all points on the vertical line upon a point at distance $R=1$.

FIG. 7.—Combined attractions upon all points on the surface by unit masses of like and unlike signs at I and III of Fig. 6.

The table shows that if unit masses at II and III have the same sign the *horizontal component*, *Fh*, for the *sum* of their attractions at $0.25R$ will be 1.95, at R it will be 1.23, which is 63 per cent of the value at $0.25R$. If the unit masses have unlike signs the *horizontal component of their difference* at $0.25R$ will be 0.93, at R it will be 0.21, which is but 23 per cent of the value at $0.25R$. The *vertical component*, *Fv*, due to the *sum* of the masses at $0.25R$ is 4.40; at R is 0.74. The *vertical component* due to the *difference* at $0.25R$ is 1.35; at R is 0.02 and of opposite sign. It is noticed that the gravity anomaly diminishes rapidly with increasing horizontal distance from these two masses and passes through zero. The deflection of the vertical first increases sharply and

then diminishes, but less rapidly than the gravity anomaly. It is important to notice that in both cases the total influence due to masses of *opposite* sign diminishes much more rapidly, and where their distance apart is 0.25 their influence is small at distance R and negligible at $2R$. This gives a means of determining whether, in the crust, anomalies and deflections are due to regional departures from isostasy or to balanced irregularities in density without absence of isostasy at the base of the zone.

To give a further illustration of balanced departures in density spread over a greater vertical distance, and representing in that way perhaps a more average case, assume that an excess or deficiency equivalent to a unit mass is at depth 0.25 and another at depth 0.75. The following tabulation shows their influence upon the surface of the earth at increasing horizontal distances.

TABLE XVIII

ATTRACTION BY UNIT MASSES AT I AND III UPON POINTS ON THE HORIZONTAL LINE

Component	Position and Sign of Mass	Horizontal Distance on Surface of Earth from Vertical Line					
		0	0.25	0.50	1.00	2.00	4.00
Fh......	− I, −III	0	−6.11	−3.56	−1.42	−0.55	−0.121
	− I, +III	0	−5.09	−2.20	−0.40	−0.03	−0.003
Fv......	− I, −III	−17.78	−7.12	−2.48	−0.61	−0.11	−0.015
	− I, +III	−14.22	−4.08	−0.40	+0.15	+0.05	+0.007

The data in this table are represented by the curves of Fig. 7. It shows that for this arrangement of masses the influence on the surface falls off rapidly at a horizontal distance between 0.25 and 0.75, which are also the vertical depths to I and III. When the masses are of opposite sign the anomaly passes through zero at a horizontal distance of about 0.6, and the deflection force for opposite sign decreases to half the value of the sum at about 0.75. The ratio between the effects of like and unlike masses becomes more marked the greater the distance of the point, although the actual magnitudes of the forces decrease.

Now assume the unit masses at I and III to be parts of masses of like density extending to the left of o to a distance N. Consider the aggregate effect upon a given point, as that at 0.50, or in general at point R. The effect of each unit at distance x to the left of o upon the point at 0.50 will be measured by an ordinate at a distance x to the right of 0.50. This will give the same aggregate result as concentrating the masses at o and summing up the area of the curve to the right of the point at 0.50 to a distance of $0.50+N$. Stated in general terms, masses at depths I and III extending linearly to distance N to the left of o will have an aggregate effect upon a point R equal to the area of the curve between R and $R+N$.

As to the aggregate effect on Fv, the gravity anomaly: If the two sheets are of negative density, it is seen that the result will be an increased negative anomaly over the effect of the separate unit masses. If the lower mass is, however, of positive density, the result for ordinarily limited sheets will be a change between o and 0.50 from a large negative to a small positive anomaly. This may be compared with the effects of other possible distributions of mass upon the gravity anomaly.

If the anomaly due to the *adjacent* departure from uniform distribution is of the mean value or greater, the *more distant* abnormal masses will have but relatively small influence. This is because the higher anomalies, with the exception of Seattle, are but two or three times the mean. Further, in a zone of large radius there are a greater number of positive and negative departures. Their aggregate effect, according to the laws of chance distribution would increase but slowly and this effect is diminished by distance according to the formula $\frac{Fv=\tan\theta\cos^3\theta}{R^2}$.

A reversal from a large anomaly of one sign to a *large* anomaly of opposite sign, rather than a *small* one of opposite sign, marks then in general a passage from an area of excess or deficiency of mass to the opposite. A gradual change in the anomaly is the reflection of a change in the subsurface abnormalities nearly as gradual. If the areal variations show that the passages of the anomaly through zero are not frequent, they go to show that limited notable irregularities of density of opposite sign in the

same column are rare. Furthermore, it has been shown under the topic "The Variable Rate of Compensation upon Gravity Anomalies" that a variable distribution of balanced densities has more effect if in areas of between 100 and 200 km. radius and has but little effect on anomalies if the balanced densities extend over much larger areas.

As to the aggregate effects produced upon *Fh*, giving deflection residuals, by these sheets I and III: If the sheets have like sign the deflection force, as shown in Fig. 7, will die out somewhat gradually and extend to considerable distances. If they have unlike sign the deflection force will fall off sharply between 0.25 and 1.00. If, however, the abnormalities of density should disappear gradually, that is, if the sheets did not terminate sharply at 0, this rate of falling off would be slower. Reversals of sign of the *deflection residuals* would require areal, not vertical, irregularities of mass. They could not take place as an effect of distance from a single mass or of two masses of unlike sign and vertically over each other. Where sharp reversals of sign take place in the deflection residuals the presence of areally contiguous areas of unlike departures in mass is shown. A mere difference in magnitude of excess of mass but of the same sign may, however, produce changes in the sign of the deflection residuals. In the irregular areal distribution of abnormal masses not balanced by being over each other, the deflection areas of like sign would thus tend to be smaller than the anomaly areas of like sign. A gradual fading-out of the deflection residuals would be the mark of gradual fading-out of the abnormal mass or the increasing influence of distant masses.

Various special combinations of three or more masses could at any one point simulate the relations indicated, but such special relations would not be of common occurrence and could not give a generality of relation of this sort.

There have thus been drawn up a set of criteria by which balanced irregularities within the zone of compensation may be distinguished from regional departures from isostasy. It remains to apply those to the areal distribution of gravity anomalies and deflection residuals as given by Hayford and Bowie. It must be recognized, however, that the stations, although numerous as

compared to previous measurements, are yet very scattered for the precise application of these tests and can at best give but qualitative results. It is thought, nevertheless, that the general nature of the answer is determinative.

GRAVITY ANOMALIES CAUSED LARGELY BY REGIONAL DEPARTURES FROM ISOSTASY

The first question is: To what degree do the areas of excess (or deficiency) of mass as indicated by gravity anomalies coincide with areas of excess (or deficiency) as shown by the deflection residuals? In Fig. 5[1] there are indicated a number of ovals shown in dot-and-dash outline and marked + or −. These are the definitely bounded areas of excess or deficiency of mass indicated by the deflection residuals. The entire surface of the crust must be constituted of such areas, but only a few are surrounded by sufficient observations to permit a boundary to be drawn at present. Even this boundary must not be regarded as sharply definite. Beside these ovals there are shown in illustrations 5 and 6, Hayford, 1909, areas of residuals characterized by like sign, referred to in the present paper as "areas of grouped residuals." They are not definitely bounded on all sides and are not shown in Fig. 5 of this article. The areas of grouped residuals show the intercepts across areas of like sign, but at least two intercepts at an angle to each other are necessary to define well the limits of the area of which they are a part. As the deflection stations are situated largely in lines or zones across the country and not surrounding the areas of like sign, it is seen why the boundaries of relatively few areas are well determined. In so far, however, as the relations of the areas of positive and negative anomaly to positive and negative deflections of the vertical are apparent, Hayford and Bowie state: "The gravity anomalies corroborate the evidence given by the deflections. In no important case are the anomalies and deflections contradictory."[2]

It is seen by inspection of the illustrations by Hayford, and also by the discussion in Part II of this article, that the areas of

[1] P. 153, Part II.

[2] Hayford and Bowie, 1912, p. 112.

like sign of deflection residuals are more sharply bounded and smaller in size than the areas of like sign of gravity anomalies. The latter occur commonly in areas so broad that a vertically balanced irregularity in the distribution of density would have but little effect. Yet the large gravity anomalies occur in the midst of such large areas, as shown on Fig. 5. There are, furthermore, few sharp reversals of sign of the gravity anomalies save those at different elevations in mountainous regions and these are explained by the presence of regional compensation. There are, on the contrary, many sharp reversals of the deflection residuals.

It is to be concluded, therefore, that, although some degree of balancing of irregularities in the same column no doubt exists, this is not a common or controlling explanation of the anomalies and residuals. They are overshadowed by a distribution which points, on the contrary, to regional departures from isostasy by regional excesses or defects in density.

In the location of stations, the deflection observations are arranged at relatively close intervals and in linear zones, owing to the necessity of triangulation. They give the most information as to the size of areas of relative excess and defect. But two areas of relative excess and defect may both be in absolute excess or absolute defect. The gravity stations are more widely scattered. The local variations are in consequence poorly defined, but the limits of absolute excess and defect of mass are determined with more accuracy. They appear to show that areas as large as 1,000 by 2,000 km., 620 by 1,240 miles, may depart in one direction from isostasy, but only to a moderate amount. It is seen from Fig. 5 that between Florida and a line drawn from Lake Superior to the Rio Grande the broad areas of less than mean anomaly are negative. From this line a great positive area extends to the northwest. The quarter of the United States bordering the Pacific Ocean is, however, another great region of negative anomalies. Upon these broad regions of mean anomaly or less are superposed smaller and better-defined areas of more than mean anomaly, negative and positive areas occurring in the same broad region. These smaller areas are inclosed by the 0.020 anomaly contour. They commonly range from 300 to 400 km. across, 200 to 250 miles, but the maxima

which reach above 0.040 are much smaller. The limits of regional isostasy appear then to vary with the amount of the load. Well-defined areas 200 to 250 miles in breadth may stand vertically 800 to 1,600 feet on the average from the level, giving isostatic equilibrium, and their central portions reach still higher values. They represent the limits of regional isostasy discussed in an earlier part. But these are superposed on broader areas which may extend for a thousand miles or more and lie as much as 400 to 800 feet either above or below the level for equilibrium. Stresses given by loads of this order are then not restricted in area to the limits set for higher values.

The size of the areas of intenser stress reveal the capacity to which the earth can carry mountain ranges uncompensated by isostasy. The size of the areas of weaker stress shows the capacity of a considerable portion of a continent to lie quiescent while the surface agencies carry forward their leveling work. This is the present state of this particular continent after a geologic period of world-wide notable vertical movement and adjustment. It is not likely, therefore, that these loads measure the maximum stress-carrying capacity of the earth. They may be more in the nature of residual stresses which the earth can hold through periods of discharge of stress. East of the Cordillera there has been but little local differential movement and these areas have lain in crustal quiet for long geologic ages, being subject only to broad and uniform crustal warping of moderate amount. It is to be presumed, therefore, that the strains which exist in such regions by virtue of the regional departures from isostasy are of ancient date and well within the limits of crustal strength.

It would seem probable for such conditions, from the standpoint of mechanics, that the zone of compensation is not sharply limited, with its implication of marked lowering of rigidity at its base; nor the distribution of compensation uniform to the base. It seems more probable that the abnormalities of density and the resultant strains should fade out through a considerable depth more after the manner suggested by Chamberlin.

[*To be continued*]